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VOLUME EQUATIONS FOR SECOND-GROWTH DOUGLAS-FIR

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ABSTRACT

This note presents volume equations and tables for second-growth Douglas-fir.

Keywords: Volume measurement (tree), Douglas-fir.

A request for a volume equation giving reasonable values for small Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco) led to a study of methods of preparing such volume tables. Although results of this study will not be reported here, we mention it to assure prospective users that the volume equations have a sound theoretical basis and, in the sense of minimizing squared residual variation, are the best fitting that could be found for the 1,127 trees in the sample.[1] The root mean square variations for both volume and form factor were the test criteria.

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[1] Cooperation of the following in providing stem analysis data is gratefully acknowledged: Weyerhaeuser Company, Canadian Forest Service, University of British Columbia, Oregon State University, Washington Department of Natural Resources, and the Intensive Culture of Douglas-fir Project of the Pacific Northwest Forest and Range Experiment Station.

The sample included trees from young stands in Oregon, Washington, and British Columbia. It included more low-elevation than high-elevation trees. Tree size ranged from 0.4 inch by 6 feet to 32 inches by 167 feet (table 1). Because of diverse sources of data, independent variables were limited to total height and diameter at breast height (b.h.) outside bark (o.b.) and inside bark (i.b.). The dependent variable was form factor, based on total cubic volume i.b. including a stump calculated as a cylinder. Stump heights were as cut, but data were collected recently so low stumps were the rule.

Two sets of equations were needed: one for very small trees and one for larger trees. There is no definite breaking point between these categories. The one chosen was 18 feet. At this height, the two equations give virtually identical volume estimates within the range of the data.

The two sets of equations follow:

H = total height in feet
D = d.o.b. at b.h. in inches
FO = outside bark form factor (FOS or FOL)
V = total volume including stump and tip in cubic feet (VS or VL)
if $H \leq 18$ feet calculate FOS (basis 59 trees)
if $H > 18$ feet calculate FOL (basis 1,068 trees)

$$FOS = 0.406098(H-0.9)^2/(H-4.5)^2 - 0.0762998\ D(H-0.9)^3/(H-4.5)^3 + 0.00262615\ DH(H-0.9)^3/(H-4.5)^3$$

$$VS = 0.005454154\ FOS(D^2H)$$

$$FOL = 0.480961 + 42.46542/H^2 - 10.99643\ D/H^2 - 0.107809\ D/H - 0.00409083\ D$$

$$VL = 0.005454154\ FOL(D^2H)$$

These equations had root mean square errors of 12.2 and 8.0 percent for FOS and FOL, respectively; 12.7 and 16.8 percent for VS and VL.

Volumes calculated from these equations appear in tables 2 and 3.

In table 2, the increases in volume with decreases in height for small trees of equal diameter result from diameter being measured 4.5 feet above ground.

For those who want to compare a logarithmic volume equation with these equations, the following had the lowest squared deviations when fit in the form stated: $\log(V-0.14) = -2.5869 + 1.0619 \log H + 1.8159 \log D$. This had a root mean square error of 12.1 percent for form factor and 18.9 percent for volume estimates.

Table 1.--Number of trees in sample

D.b.h. (inches)	Height (feet)																	Total
	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	
<0.5	1																	1
1	34	8																42
2	10	49	6															65
3		16	54	5														75
4			32	11	2													45
5		1	14	25	10	3	1		1									55
6			1	32	15	14	15	6										83
7			2	13	18	10	19	15	3									80
8				2	21	3	22	26	3	2								79
9				3	12	8	8	34	15	5	1							86
10					1	6	9	23	17	10		2						68
11				1	1	2	10	11	22	12	3	2						64
12						1	3	9	18	22	5	2	1	1				62
13							3	7	6	21	14	6	1					58
14							1	4	9	16	9	7	1	2				49
15							1	1	10	6	4	4	3	2	1			32
16								1	2	6	8	7	6	3	1			34
17									1	2	4	4	3	2	1			17
18									1	2		5	5	2		1		16
19									1	5	3	1	5	3	2			20
20											2	2	2	3		3		12
21										2	2	3	6	3	6	1		23
22										1	2	5	2	2		3		15
23										2			1		2	2	3	10
24												2		1		4		7
25												1		2		1	1	5
26										1	1	1		1	3	1		8
27											2			3			1	6
28												1		1	1			3
29																1	1	2
30														3	1			4
31																		
32																	1	1
Total	45	74	109	92	80	47	92	137	109	115	60	55	36	34	18	17	7	1,127

Table 2.--Estimated cubic-foot volume, Douglas-fir under 18 feet in height, including stump and tip--basis 59 trees

D.b.h. (inches)	Height (feet)												
	6	7	8	9	10	11	12	13	14	15	16	17	18
.4	0.020	0.013											
.5	.029	.019	0.016	0.014	0.014								
.6	.038	.026	.022	.020	.019								
.7	.049	.034	.029	.026	.025	0.025	0.025	0.026					
.8	.058	.043	.036	.034	.032	.032	.032	.033	0.034	0.035			
.9	.068	.051	.044	.041	.040	.040	.040	.041	.042	.043	0.045	0.046	
1.0		.060	.053	.050	.048	.048	.049	.050	.051	.053	.054	.056	
1.1		.069	.061	.058	.057	.057	.058	.059	.061	.063	.065	.067	0.070
1.2			.070	.067	.066	.066	.067	.069	.071	.074	.076	.079	.082
1.3			.079	.076	.075	.076	.078	.080	.082	.085	.088	.092	.095
1.4				.085	.085	.086	.088	.091	.094	.097	.101	.105	.109
1.5				.094	.095	.096	.099	.102	.106	.110	.115	.119	.124
1.6					.105	.107	.110	.114	.119	.123	.129	.134	.140
1.7					.115	.118	.122	.126	.132	.137	.143	.150	.156
1.8					.124	.128	.133	.139	.145	.151	.158	.166	.173
1.9					.134	.139	.145	.151	.159	.166	.174	.182	.191
2.0						.150	.157	.164	.173	.181	.190	.200	.210
2.1						.161	.169	.177	.187	.196	.207	.217	.228
2.2							.181	.191	.201	.212	.224	.236	.248
2.3							.192	.204	.216	.228	.241	.254	.268
2.4								.217	.230	.244	.258	.273	.288
2.5								.230	.245	.260	.276	.292	.309
2.6									.259	.276	.294	.312	.331
2.7									.274	.293	.312	.332	.352
2.8										.309	.330	.352	.374
2.9										.326	.349	.372	.397
3.0											.367	.393	.419
3.1											.385	.413	.442
3.2											.404	.434	.465
3.3												.455	.488
3.4												.476	.512
3.5												.496	.535

Table 3.--Estimated cubic-foot volume, Douglas-fir, over 18 feet in height (including stump and tip)--basis 1,068 trees

D.b.h. (inches)	Height (feet)																
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
1	0.06																
2	.22	0.32															
3	.47	.69	0.92														
4	.77	1.17	1.58	1.99	2.39	2.80	3.20	3.61									
5	1.10	1.75	2.40	3.03	3.67	4.30	4.93	5.56	6.19								
6	1.43	2.41	3.34	4.26	5.17	6.08	6.98	7.89	8.79								
7	1.75	3.12	4.40	5.66	6.90	8.13	9.35	10.6	11.8								
8		3.86	5.56	7.20	8.82	10.4	12.0	13.6	15.2	16.8							
9		4.63	6.80	8.88	10.9	13.0	15.0	17.0	18.9	20.9	22.9						
10		5.39	8.09	10.7	13.2	15.7	18.2	20.6	23.0	25.5	27.9	30.3					
11			9.43	12.6	15.6	18.6	21.6	24.5	27.5	30.4	33.3	36.2	39.1				
12			10.8	14.5	18.2	21.7	25.2	28.7	32.2	35.7	39.1	42.5	46.0	49.4			
13				16.6	20.8	25.0	29.1	33.2	37.2	41.2	45.2	49.3	53.3	57.2			
14					23.6	28.4	33.1	37.8	42.5	47.1	51.7	56.4	61.0	65.5			
15					26.4	31.9	37.3	42.7	48.0	53.3	58.6	63.8	69.0	74.2	79.5		
16						35.5	41.7	47.7	53.7	59.7	65.7	71.6	77.5	83.4	89.3		
17						39.2	46.1	53.0	59.7	66.4	73.0	79.7	86.3	92.9	99.5		
18							50.7	58.3	65.8	73.3	80.7	88.0	95.4	103	110	117	
19							55.4	63.8	72.1	80.3	88.5	96.7	105	113	121	129	
20							60.1	69.4	78.5	87.6	96.6	106	114	123	132	141	150
21								75.0	85.0	95.0	105	115	124	134	144	153	163
22								80.8	91.7	102	113	124	134	145	156	166	177
23								86.6	98.4	110	122	133	145	156	168	179	190
24								92.4	105	118	130	143	155	168	180	192	204
25								98.2	112	126	139	153	166	179	192	206	219
26								104	119	133	148	162	177	191	205	219	233
27									126	141	157	172	187	203	218	233	248
28										149	166	182	198	215	231	247	263
29											175	192	209	226	244	261	278
30												202	220	238	257	275	293
31													231	250	270	289	308
32													242	262	283	303	323
33														274	296	317	338

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